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DYNAMICS OF RETRIEVAL OF A SPACE TETHERED SYSTEM†

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The motions of a tethered system in the retrieval process are investigated using a simple non-linear model. The unique motion of the system at a constant tether retrieval rate, in which the angle of deviation of the tether vanishes at the end of the retrieval, is found. A method of controlling the retrieval process is proposed, which prevents the amplitude of oscillations increasing.

The dynamics of space tethered systems have attracted increasing attention in recent years, in view of the possible use of such systems in various applications [1, 2]. The essential problem is to construct a rational control of the tether retrieval process, since the tether may oscillate at increasing amplitudes while being retrieved.

1. EQUATIONS OF MOTION

Let us consider the motion of a satellite-probe S of mass m, linked by a tether to a spacecraft K moving in a circular orbit of radius R_0 around the Earth (see Fig. 1). It is assumed that the mass of K is significantly greater than that of the satellite, and that the position and orientation of the spacecraft are rigidly stabilized in the orbital frame of reference. The tether is assumed to be absolutely flexible and inextensible, and of mass much less than the satellite mass m. The linear dimensions of the satellite are small compared with the length of the tether, so that the satellite may be considered as a point mass m. We shall confine our attention to plane motions of the tethered satellite system in the plane of the circular orbit of the spacecraft K.

Let $Oxyz$ be a Cartesian system of coordinates rigidly attached to the spacecraft K, with origin O at the point of the spacecraft from which the tether issues, the x-axis along the local vertical to the Earth's centre C, the y-axis along the spacecraft's velocity vector and the z-axis along the normal to the orbital plane (Fig. 1). Let $\mathbf{R}_0$ and $\mathbf{R}$ denote the radius-vectors of the points O and S, respectively, relative to C. Then the vector $\mathbf{r} = \mathbf{R} - \mathbf{R}_0$, which points along the tether OS, is the radius-vector of the satellite S in $Oxyz$ coordinates.

Let θ denote the angle between $\mathbf{r}$ and the x-axis. Then the vector $\mathbf{r}$, the velocity $\mathbf{v} = \mathbf{r}^{\cdot}$ and the acceleration $\mathbf{w} = \mathbf{v}^{\cdot} = \mathbf{r}^{\cdot\cdot}$ of the satellite S in the frame of reference $Oxyz$ have the following components

$$
\begin{aligned}
&\mathbf{r} = (r\cos\theta,\ r\sin\theta,\ 0)\\
&\mathbf{v} = (r^{\cdot}\cos\theta - r\theta^{\cdot}\sin\theta,\ r^{\cdot}\sin\theta + r\theta^{\cdot}\cos\theta,\ 0)\\
&\mathbf{w} = (r^{\cdot\cdot}\cos\theta - 2r^{\cdot}\theta^{\cdot}\sin\theta - r\theta^{\cdot\cdot}\sin\theta - r(\theta^{\cdot})^2\cos\theta\\
&r^{\cdot\cdot}\sin\theta + 2r^{\cdot}\theta^{\cdot}\cos\theta + r\theta^{\cdot\cdot}\cos\theta - r(\theta^{\cdot})^2\sin\theta,\ 0)
\end{aligned}
\tag{1.1}
$$

The forces acting on S in the frame of reference $Oxyz$ are: the tension $\mathbf{T}$ in the tether, the gravitational force $\mathbf{F}_1$, and the inertial forces—the centrifugal force $\mathbf{F}_2$ and the Coriolis force $\mathbf{F}_3$. The latter three forces are defined by the formulae

$$
\mathbf{F}_1 = -m\Omega^2 R_0^3 R^{-3}\mathbf{R}, \quad \mathbf{F}_2 = m\Omega^2\mathbf{R}, \quad \mathbf{F}_3 = -2m\Omega(\mathbf{n}\times\mathbf{v}) \tag{1.2}
$$

where Ω is the angular velocity of revolution of the spacecraft K in orbit and $\mathbf{n}$ is the unit normal to the orbital plane, which points along the z-axis.

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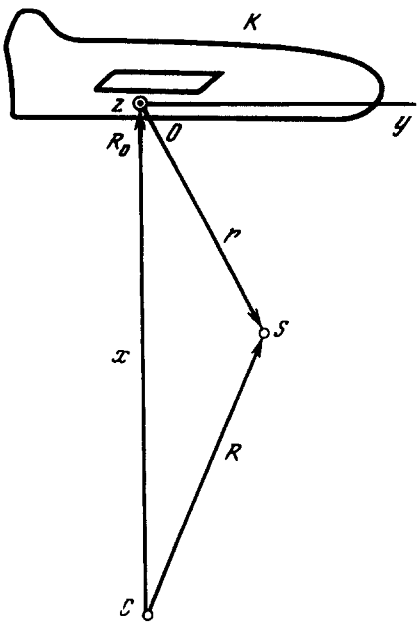

Fig. 1.

To simplify formulae (1.2), we substitute $\mathbf{R} = \mathbf{R}_0 + \mathbf{r}$ throughout and bear in mind that the tether length is small compared with the radius of the orbit: $r \ll R_0$. Noting that in the frame of reference $Oxyz$ we have $\mathbf{R}_0 = (-R_0, 0, 0)$, we obtain

$$\mathbf{F}_1 + \mathbf{F}_2 = m\Omega^2\mathbf{R}[1 - R_0^{-3}(R_0^2 + 2\mathbf{R}_0\cdot\mathbf{r} + r^2)^{-3/2}] = \tag{1.3}$$
$$= 3m\Omega^2 R_0^{-2}(\mathbf{R}_0\cdot\mathbf{r})R_0 = (3m\Omega^2 r\cos\theta, 0, 0)$$

To simplify the expression for $\mathbf{F}_3$ in (1.2), we substitute the components of the vector $\mathbf{v}$ from (1.1)

$$\mathbf{F}_3 = 2m\Omega(r^{\cdot}\sin\theta + r\theta^{\cdot}\cos\theta,\ -r^{\cdot}\cos\theta + r\theta^{\cdot}\sin\theta,\ 0) \tag{1.4}$$

We can now set up the equation of motion of S in $Oxyz$ coordinates. The first equation follows from the theorem determining the variation of the momentum of S relative to the pole O

$$m(r^2\theta^{\cdot})^{\cdot} = [\mathbf{r}\times(\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3)]\cdot\mathbf{n}$$

We substitute (1.3) and (1.4) into this equation, as well as the components of the vectors $\mathbf{r}$ from (1.1) and $\mathbf{n} = (0, 0, 1)$ and introduce dimensionless time—the true anomaly—by $\nu = \Omega t$. Changing to the new independent variable ν and simplifying, we get

$$\frac{d^2\theta}{d\nu^2} + \frac{2}{r}\frac{dr}{d\nu}\left(\frac{d\theta}{d\nu} + 1\right) + 3\sin\theta\cos\theta = 0 \tag{1.5}$$

To derive the second equation we project the vector equation of motion of S in $Oxyz$ coordinates onto the direction of $\mathbf{r}$

$$m\mathbf{w}\cdot\mathbf{e} = (\mathbf{T} + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3)\cdot\mathbf{e},\quad \mathbf{e} = (\cos\theta, \sin\theta, 0) \tag{1.6}$$

The tether tension $\mathbf{T}$ applied to the satellite is in the direction opposite to that of the unit vector $\mathbf{e}$: $\mathbf{T} = -T\mathbf{e}$. Substituting the formula for $\mathbf{w}$ from (1.1) into (1.6), as well as (1.3) and (1.4), we obtain

$$T = m\Omega^2 r\Phi,\quad \Phi = 3\cos^2\theta + 2\theta'_\nu + (\theta'_\nu)^2 - r^{-1}r''_{\nu\nu} \tag{1.7}$$

For the tether to remain taut, as is assumed in this paper, it is necessary and sufficient that $T \geqslant 0$, or $\Phi \geqslant 0$.

Formulae (1.5) and (1.7), as well as equations for the dynamics of a space tethered system in a more general case, were derived in [1].

2. THE RETRIEVAL PROCESS

Qualitative studies and numerical computations show that when the tether is being retrieved ($dr/d\nu \leqslant 0$), the amplitude of the oscillations generally increases rapidly, in view of the singularity of Eq. (1.5) as $r \to 0$. It may also happen that oscillatory motions with respect to θ will become rotations. As such motions are, of course, inadmissible, the problem arises of controlling the retrieval process in such a way that the angle of deviation θ of the tether remains bounded as $r \to 0$.

There are different ways of controlling the retrieval process: (1) small motors can be installed on the satellite S in order to generate reactive forces perpendicular to the tether; such forces will make it possible to prevent large deviations of the tether from the vertical, (2) the point of suspension O of the tether may be set in motion relative to the body of the spacecraft K; suitably chosen, such motions will suppress tether oscillations, (3) the spacecraft itself may deviate slightly from the chosen circular orbit; in terms of the dynamics of the system, this method is similar to the second, and (4) the tension of the tether during retrieval may be controlled; this method was studied in [2], in a linearized formulation of the problem.

In this paper we will consider the possibility of controlling the retrieval process by suitably regulating the variation of the tether length $r(\nu)$. In other words, the law governing the winding of the tether is such that the angle θ the tether makes with the vertical will remain bounded or tend to zero as $r \to 0$. It will be assumed that the rate of winding $u = -r^{\cdot}$ during the retrieval process may take one of two constant values $u = 0$ or $u_0 > 0$. Thus, at each instant of time the tether either maintains a constant length or is retrieved at a constant rate u_0. This condition imposes fairly simple and practicable demands on the winding equipment.

In order to construct a suitable retrieval routine, we shall first make a separate study of the passive periods, during which the tether length remains constant and no winding is occurring ($u = 0$, $r =$ const), and of the active periods, during which there is a constant [non-zero] rate of winding ($dr/dt = -u_0 < 0$).

3. CASE OF CONSTANT TETHER LENGTH

If the tether length is constant ($r =$ const), Eq. (1.5) becomes the equation of a pendulum

$$d^2(2\theta)/d\nu^2 + 3\sin 2\theta = 0 \tag{3.1}$$

which can be integrated in terms of elliptic functions. Equation (3.1) has the following first integral (the energy integral)

$$\omega^2/2 + {}^3/_4(1 - \cos 2\theta) = h, \quad \omega = d\theta/d\nu \tag{3.2}$$

where ω is the dimensionless angular velocity of the tether oscillations and $h \geqslant 0$ is the energy constant. If $h = 0$ we have a stable equilibrium $\theta = k\pi$ ($k = 0, \pm 1, \ldots$), $\omega = 0$; values $h \in (0, {}^3/_2)$ correspond to oscillations of the tether, while $h > {}^3/_2$ means that the tether rotates about the point of suspension.

Without loss of generality, we shall assume, to fix our ideas, that the system should be brought to the stable state $\theta = 0$, $\omega = 0$. At $h = {}^3/_2$ we have aperiodic motions along separatrices in the phase space that pass through saddle-type singular points (unstable equilibria $\theta = \pm\pi/2$). In these aperiodic motions, by (3.2)

$$\omega = \pm 3^{1/2} \cos\theta \tag{3.3}$$

The phase trajectories of the oscillatory and aperiodic motions at constant cable length are represented in Fig. 2 by thin curves. The arrows indicate the direction of motion. The oscillatory region is bounded by two phase trajectories of aperiodic motions (3.3). Outside this region one has rotations.

The value of Φ from (1.7), for motions at constant tether length ($r =$ const), is

$$\Phi = 3\cos^2\theta + 2\omega + \omega^2 \tag{3.4}$$

In the region where $\Phi \geqslant 0$ the tether remains taught; but in regions where $\Phi < 0$ it slackens, and the

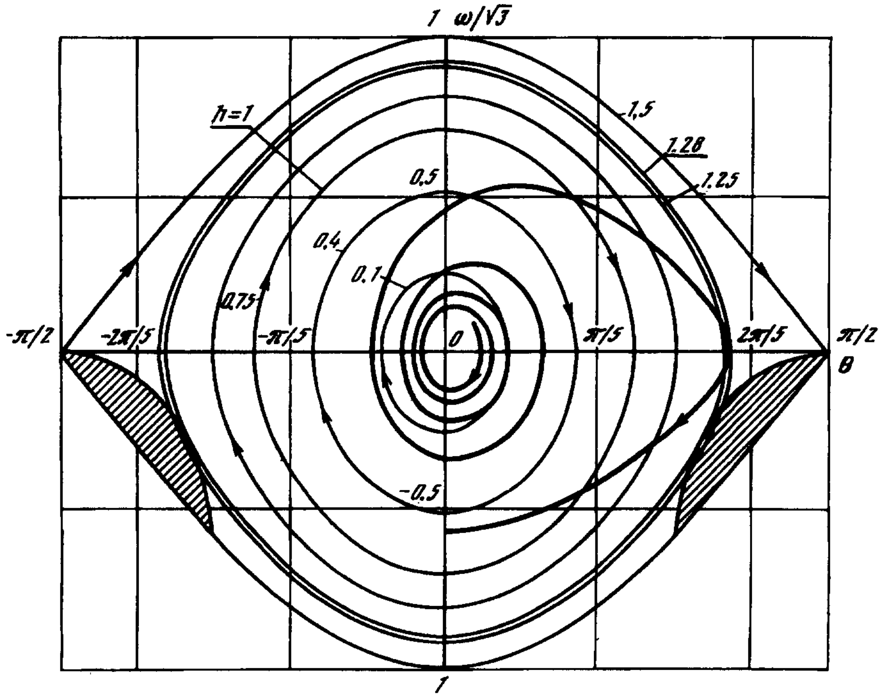

Fig. 2.

constraint realized by the tether becomes non-active. The regions $\Phi < 0$ are shown hatched in Fig. 2. Computing the minimum of h as defined in (3.2), on condition that $\Phi = 0$, we find the coordinates of the points at which it is achieved and the minimum itself

$$\theta = \pm\pi/3, \quad \omega = -{}^1/_2, \quad h = {}^5/_4 \tag{3.5}$$

At these points the curve $\Phi = 0$ touches the phase trajectory of non-linear oscillations corresponding to $h = 1.25$. At $h \leqslant 1.25$ the constraint remains active and the tether remains taut [1].

4. OSCILLATIONS AT A CONSTANT RATE OF WINDING

At a constant rate of winding the tether length is a linear function of time

$$r = r_0 - u_0 t = r_0 - u_0\Omega^{-1}\nu \quad (\nu = \Omega t) \tag{4.1}$$

where r_0 is the initial length of the tether at time $t = 0$ and $u_0 \geqslant 0$ is the constant rate of winding.

Let us fix the zero of the dimensionless time τ so that the tether length is zero at time $\tau = 0$. To do this, we put

$$r_0 = -u_0\Omega^{-1}\tau, \quad \tau = -\Omega u_0^{-1} r = \nu - r_0 u_0^{-1}\Omega \tag{4.2}$$

where τ is the new dimensionless time, which coincides apart from sign with the dimensionless tether length. Substituting formulae (4.2) into Eq. (1.5), we obtain

$$\theta''_{\tau\tau} + 2\tau^{-1}(\theta'_t + 1) + {}^3\!/_2 \sin 2\theta = 0 \tag{4.3}$$

The argument τ in (4.3) takes only non-positive values: $\tau \leqslant 0$. Note that at a constant rate of winding, as in the case of constant tether length, $r''_{\nu\nu} = 0$, and the value of Φ from (1.7) again reduces to the form (3.4). Therefore the regions of the phase plane in which $\Phi < 0$ and the constraint becomes non-active are given, as before, by the hatched regions in Fig. 2 (see Section 3).

5. SMALL OSCILLATIONS OF THE TETHER

Let us first consider the case of small oscillations $|\theta| \ll 1$ at a constant rate of winding. In such cases Eq. (4.3) can be linearized, by replacing the last term on the left by 3θ.

The general solution of the linearized equation is [1]

$$\theta = \tau^{-1}[A\cos(\sqrt{3}\tau) + B\sin(\sqrt{3}\tau) - \tfrac{2}{3}] \tag{5.1}$$

where A and B are arbitrary constants.

A particular solution that suppresses the oscillations at the end of the retrieval process ($\theta \to 0$ as $\tau \to -0$) is

$$\theta = \tfrac{2}{3}\tau^{-1}[\cos(\sqrt{3}\tau) - 1] \tag{5.2}$$

The absolute value of this solution reaches its maximum at $\tau = -1.34$, this maximum being 0.841. At such amplitudes of the oscillations, the use of a linearized approach based on the assumption $|\theta| \ll 1$ cannot be considered to be correct.

6. NON-LINEAR OSCILLATIONS OF THE TETHER

Let us return to the non-linear equation (4.3) and find a particular solution such that

$$\theta \to 0, \quad \theta'_\tau \to \text{const} \quad \text{as} \quad \tau \to -0 \tag{6.1}$$

This means that the angle between the tether and the vertical must vanish at the end of the retrieval process, while the angular velocity should remain finite. We shall seek such a solution for small τ in the form of the expansion

$$\theta^*(\tau) = \sum_{i=1}^{\infty} \theta_i \tau^i \tag{6.2}$$

where θ_i are undetermined coefficients. Note that the solution is an odd function of θ, so that all even-numbered coefficients vanish. Substituting the expansion (6.2) into Eq. (4.3) and equating the coefficients of successive powers of τ to zero, we obtain

$$\theta^*(\tau) = -\tau + \tau^3/4 - 11\tau^5/120 + O(\tau^7) \tag{6.3}$$

Differentiating (6.3), we obtain the expansion of the angular velocity $\omega = d\theta/d\tau$

$$\omega^*(\tau) = -1 + 3\tau^2/4 - 11\tau^4/24 + O(\tau^6) \tag{6.4}$$

To eliminate τ from (6.3) and (6.4), we expand $\omega = \omega(\theta)$ in a power series with undetermined coefficients; the coefficients are determined by substituting (6.3) and (6.4) into the expansion and equating coefficients of like powers of τ. The result is

$$\omega = -1 + 3\theta^2/4 - \theta^4/12 + O(\theta^6) \tag{6.5}$$

Formulae (6.3) and (6.4) determine the required solution as $\tau \to -0$, i.e. at the end of the tether retrieval process. Equation (6.5) defines the corresponding segment of the phase trajectory.

We must still determine expansions for h and Φ. Substituting (6.3) and (6.4) into (3.2) and (3.4), we get

$$\begin{aligned} h^*(\tau) &= \omega^2/2 + 3(1-\cos 2\theta)/4 = \tfrac{1}{2} + 3\tau^2/4 - 49\tau^4/96 + O(\tau^6) \\ \Phi^*(\tau) &= 3\cos^2\theta + 2\omega + \omega^2 = 2 - 3\tau^2 + 49\tau^4/16 + O(\tau^6) \end{aligned} \tag{6.6}$$

A solution $\theta^*(\tau)$ of the non-linear equation (4.1) satisfying conditions (6.1) was constructed by numerical means. Integration was performed in reverse time, using the expansions (6.3) and (6.4) to determine the initial data for small negative τ. Curves of $\theta^*(\tau)$ and $\omega^*(\tau)$ obtained by numerical integration are shown in Fig. 3. They are bounded functions for all $\tau \leq 0$, with $\theta^* \to 0$ and $\omega^* \to 0$ as $t \to -\infty$. The phase trajectory $\omega(\theta)$ for the solution constructed is indicated by the thickened curve in Fig. 2. It lies entirely in the region of oscillatory motions $h < 1.5$. Along the whole trajectory we have the strict inequality $\Phi > 0$ for all $\tau \leq 0$, so that the tether remains taught at all times. This is also evident from Fig. 2: the phase trajectory of the solution lies outside the hatched regions, where $\Phi < 0$. The thick curve in Fig. 4 is a plot of $h = h^*(\tau)$ for the solution $\theta^*(\tau)$, computed using (3.2).

The solution $\theta^*(\tau)$ is essentially different from the solution (5.2) of the linearized equation. Thus, the deviation of the tether in the solution $\theta^*(\tau)$ takes the value $\theta = 1.167$ at $\tau = 2.5$; this value lies outside the region in which linear approximation is admissible.

Note that instead of conditions (6.1) one could have postulated the more general conditions

$$\theta \to \delta, \quad \theta'_\tau \to \text{const} \quad \text{as} \quad \tau \to -0 \tag{6.7}$$

where δ is a given constant. A solution of Eq. (4.3) satisfying conditions (6.7) may also be sought as an expansion analogous to (6.2).

$$\theta_\delta(\tau) = \delta + \sum_{i=1}^{\infty} \theta_i \tau^i \tag{6.8}$$

The undetermined coefficients θ_i in (6.8) are found in the same way as those for the solution (6.2). The first few coefficients are

$$\begin{aligned} &\theta_1 = -1, \quad \theta_2 = -{}^1\!/\!_4 \sin 2\delta, \quad \theta_3 = {}^1\!/\!_4 \cos 2\delta, \\ &\theta_4 = 3\sin 2\delta(4 + \cos 2\delta)/80, \quad \theta_5 = (6 - 8\cos 2\delta - 9\cos^2 2\delta)/120 \end{aligned} \tag{6.9}$$

If $\delta = 0$, these formulae yield the corresponding coefficients in (6.3). A solution satisfying (6.9) may be treated in the same way as $\theta^*(\tau)$.

Note that the solution $\theta^*(\tau)$ has the following property: the corresponding $h^*(\tau)$ is the minimal function as $\tau \to -0$ with respect to all functions $h(\tau)$ corresponding to solutions of Eq. (4.3).

Indeed, for solutions of Eq. (4.3) that are unbounded as $\tau \to -0$ one has $\omega \to \infty$, $h \to \infty$ as $\tau \to -0$. For solutions that are bounded for $\tau \to -0$ it follows from (6.7), (6.8) and (3.2) that

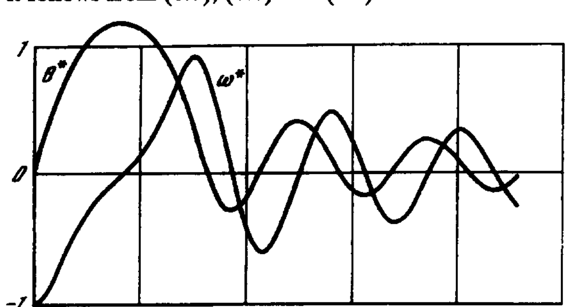

Fig. 3.

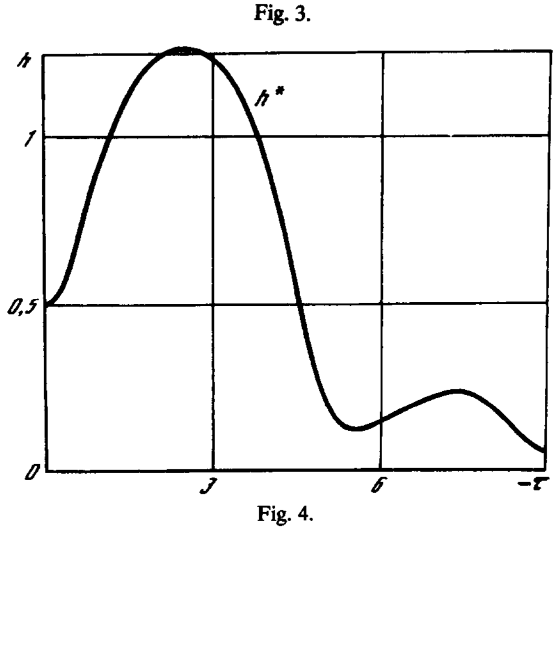

Fig. 4.

$$\omega \to -1, \quad \theta \to \delta, \quad h \to {}^1/2 + 3(1 - \cos 2\delta)/4 \quad \text{as} \quad \tau \to -0 \tag{6.10}$$

Comparing relationships (6.6) and (6.10), we see that $h \geqslant h^*$ for $\tau \to -0$, which it was required to prove.

7. CONTROL OF THE RETRIEVAL PROCESS

Let the initial data be

$$t = 0, \quad r = r_0, \quad \theta = \theta_0, \quad \theta^{\cdot} = \theta^{\cdot}_0$$

Changing to the true anomaly $\nu = \Omega t$ and using the notation $\omega = \theta'_\nu$ from (3.2), we get

$$\nu = 0, \quad r = r_0, \quad \theta = \theta_0, \quad \omega = \omega_0 = \theta^{\cdot}_0 \Omega^{-1} \tag{7.1}$$

Let us assume that the phase state (θ_0, ω_0) lies within the region of closed trajectories in the phase plane (θ, ω) (Fig. 2).

Using the solution given in Section 6, we can propose some ways of controlling the retrieval process. The basic idea is to proceed from the initial state (7.1) to one of the solutions of Section 6 corresponding to bounded oscillations. We shall consider two possible approaches.

1. Beginning at time $\nu = 0$, wind in the tether at a constant rate u_0, chosen subject to the condition that the solution is identical with the particular solution $\theta_\delta(\tau)$ of (6.8). The initial conditions (7.1) may be rewritten as

$$\theta(\tau_0) = \theta_0, \quad \omega(\tau_0) = \omega_0 \tag{7.2}$$

where τ_0 is the initial value of τ, which may be expressed by (4.2) in terms of the rate of winding u_0

$$\tau_0 = -r_0 u_0^{-1} \Omega \tag{7.3}$$

To determine the two unknown parameters—the parameter δ of the solution $\theta_\delta(\tau)$ of (6.8) and the parameter τ_0—we have two conditions (7.2). In other words, the required $\theta_\delta(\tau)$ is determined by solving a two-point boundary-value problem for Eq. (4.3). At one end of the interval (at the singular point $\tau = 0$) we have conditions (6.7), which determine a one-parameter solution (6.8) with parameter δ; at the other, for a non-fixed value of the argument $\tau = \tau_0 < 0$, we have two conditions (7.2). After solving this problem and determining the parameters δ and τ_0, one finds the rate of winding from (7.3): $u_0 = -r_0\Omega\tau_0^{-1} > 0$. The total retrieval time equals $T = -\Omega^{-1}\tau_0 = r_0 u_0^{-1}$.

This control method is satisfactory if the rate of winding u_0 of (7.3) lies within the admissible limits and the tether remains taut when governed by the solution $\theta = \theta_\delta(\tau)$, that is, $\Phi \geqslant 0$. These questions, as well as those of the existence and uniqueness of a solution of the problem, require additional attention.

2. The retrieval process is carried out in two stages. At the first stage, the tether maintains a constant length $r = r_0$ for $\nu \in [0, \nu^*]$. During that time the motion of the system proceeds in accordance with one of the closed phase trajectories for $r = \text{const}$, illustrated in Fig. 2. At a certain time $\nu = \nu^*$, the phase trajectory, beginning at the point (θ_0, ω_0) and described by functions $\theta(\nu)$, $\omega(\nu)$, intersects the phase trajectory of the solution $\theta^*(\tau)$ depicted by the thick curve in Fig. 2. At that instant one has

$$\theta(\nu^*) = \theta^*(\tau^*), \quad \omega(\nu^*) = \omega^*(\tau^*) \tag{7.4}$$

where τ^* is the value of the parameter τ in the solution $\theta^*(\tau)$, $\omega^*(\tau)$ at the time of intersection. For each point of intersection (of which there may be several, see Fig. 2) there is a definite $\tau = \tau^*$. At time $\nu = \nu^*$ the cable length is r_0 and is expressed in terms of τ^* by (4.2). Hence we can determine the necessary constant rate of winding in the second stage of the motion

$$u_0 = -r_0\Omega(\tau^*)^{-1} > 0 \tag{7.5}$$

The second stage begins at time $\nu = \nu^*$ and proceeds at the constant rate of winding u_0 until the retrieval process ends. Here the motion is described by the solution $\theta^*(\tau)$ of Section 6; it ends when conditions (6.1) hold. The total time of motion in this retrieval process equals

$$T = \Omega^{-1}(\nu^* - \tau^*) \tag{7.6}$$

The initial value h_0 of the energy is determined by (3.2).

If $h \leqslant 1.28$, the second control method will always be feasible, since any closed phase trajectory of Fig. 2 will intersect the phase trajectory $(\theta^*(\tau), \omega^*(\tau))$ at least once. This may happen at more than one point (usually at two), implying different values of the parameter τ^* and, by (7.5), different rates of winding of the tether in the second stage. Of all these possibilities, one can select the one with the most satisfactory rate of winding.

Note that the constraint is active in both stages of the motion, provided that $h_0 \leqslant 1.25$. In the narrow range $1.25 < h_0 < 1.28$, for certain initial data (θ_0, ω_0), the constraint may become non-active over very small intervals of the motion; this is apparently not critical. The point is that the parts of the regions $\Phi < 0$ between the close trajectories with $h = 1.25$ and $h = 1.28$ form very small lunes within which the breakdown of the condition $\Phi \geqslant 0$ is, moreover, insignificant.

At the boundary between the first and second stages (at $\nu = \nu^*$) the rate of winding jumps from $u = 0$ to $u = u_0 > 0$, so that in the neighbourhood of $\nu = \nu^*$ we have

$$r''_{\nu\nu} = -u_0\Omega^{-1}\delta(\nu - \nu^*) \tag{7.7}$$

where δ is the delta-function. Substituting (7.7) into (1.7), we see that at time $\nu = \nu^*$ the tether experiences an impact load. It is natural to replace this jumpwise change of the rate of winding by a smooth increase, representing the action of the motor of the winding equipment. This smooths out the jump and leads to a substantial decrease in tension.

Note that the methods proposed above for controlling the winding of the tether were of the open-loop type, based on the simplest mechanical model of an inextensible and weightless tether oscillating in the orbital plane. Many important factors remain outside the scope of this approach: allowance for the inertia and elasticity of the tether, which may generate longitudinal and transverse oscillations, as well as breakdown of the constraint, initial deviations of the tether perpendicular to the orbital plane, and the existence of disturbances and measurement errors. Such factors require the application of closed-loop control, so that the methods proposed above need substantial adjustments. Nevertheless, it seems that at the final step of the retrieval process the essentially non-linear solutions constructed here, which remain bounded as $r \to 0$, may be used to select basic (open-loop) motions.

8. EXAMPLES

We present a few examples of numerical simulation of the tether retrieval process. The initial data were set in the dimensionless form (7.1). The retrieval process was based on the second approach described in Section 7. We first constructed the solution $\theta^*(\tau)$, $\omega^*(\tau)$ of Section 6 (see Figs 2 and 3).

At the first stage Eq. (3.1) was integrated with initial data (7.1). The first stage ended when conditions (7.4) were satisfied. From these conditions we determined the values of the parameters $\nu^* > 0$ and $\tau^* < 0$, after which the second stage began, the solution being determined by the functions $\theta^*(\tau)$, $\omega^*(\tau)$ for $\tau \in [\tau^*, 0]$. The rate of winding at the second stage, u_0, and the total time of motion T are given by (7.5) and (7.6), respectively.

A few results of the computations for four versions are shown below

N	1	2	3	4
θ_0	−0.5	0	0.75	0.4
ω_0	0	−0.9	0.75	−0.6
ν^*	5.0	4.9	4.15	4.9
$-\tau^*$	0.94	2.03	0.01	2.53

and in Fig. 5; the digits on the curves indicate the different versions and the dots indicate the boundaries of the stages.

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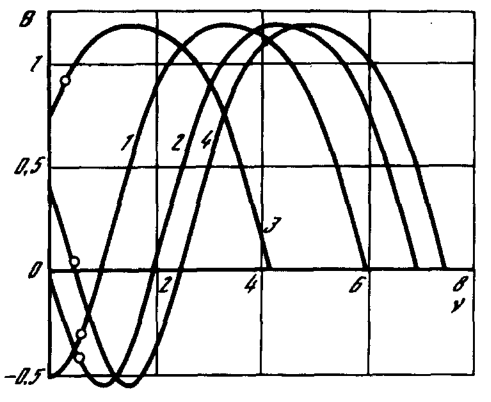

Fig. 5.

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